INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & MANAGEMENT

Arbitrary amplitude ion acoustic waves in finite temperature Fermi plasma

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Abstract

Arbitrary amplitude solitary waves are investigated in finite temperature quantum plasma containing electrons and ions by employing Sagdeev's pseudopotential technique. The effect of electron degeneracy, ion temperature, quantum diffraction has important contribution in determining the nature of pseudopotential well. They also determine the formation and properties of ion acoustic waves in this two-component electron-ion dense quantum plasma.

Key words- quantum plasmas, quantum hydrodynamic model, finite temperature effect, ion-acoustic waves, Sagdeev's pseudopotential method, solitary waves

I. INTRODUCTION

In the recent years, quantum plasma has been a major field of interest. It has applications ranging from space plasmas (white dwarf, neutron stars etc.) [1] to laboratory produced plasmas. It has found applications in biophotonics [2], microelectronics [3], carbon nanotubes [4], laser-solid interactions [5], metal nanostructures [6], etc. Therefore, the surge in research in the field of quantum plasma is justified. Traditionally, there has been research in quantum effects in plasmas [7-9]. Recent studies by Haas [10], Shukla [11], Manfredi [12], Brodin, Marklund [13, 14], Eliasson [15], Misra [16], Chatterjee [17, 18], Ghosh [19, 20], Chandra [21-31], Roychoudhury [32], Sahu [33] have made major contributions in this field. Most of them have used the quantum hydrodynamic (QHD) model. The hydrodynamic model makes use of the fluid equations in studying the plasma considering it to be as a fluid. Quantum plasma is characterized by high density and low temperature, in contrast to the classical plasma. What distinguishes quantum plasma from its classical counterpart is the coupling parameter. The coupling parameter is defined as the ratio of the potential to the kinetic energy. For both classical and quantum Coulomb systems, the mean interaction energy U_{pot} is the same and is given by

$$U_{pot} = \frac{e^2 n_0^{1/3}}{\varepsilon_0} \tag{1}$$

But the mean kinetic energies differ. In the classical and quantum case they are given as:

$$K_C = k_B T \tag{2}$$

$$K_Q = k_B T_F \tag{3}$$

Here T_F is the Fermi temperature given by:

$$T_F = \frac{\hbar^2}{2m} \cdot \frac{(3\pi^2 n)^{2/3}}{k_B} \tag{4}$$

The classical and quantum coupling parameters are given by:

$$\Gamma_C = \frac{U_{pot}}{K_C} = \frac{e^2 n_0^{\frac{1}{3}}}{\varepsilon_0 k_B T} \sim \left(\frac{1}{n_0 \cdot \lambda_D{}^3}\right)^{\frac{1}{3}} = 2.1\times10^{-7}\times\frac{n_0^{\frac{1}{3}}}{T} \tag{5}$$

$$\Gamma_Q = \frac{U_{pot}}{K_Q} = \frac{2me^2\pi^{-4/3}}{\pi^{4/3}\varepsilon_0\hbar^2 n_0^{1/3}} \sim \left(\frac{1}{n_0 \lambda_F{}^3}\right)^{\frac{2}{3}} = \frac{10^{11}}{2} n_0^{-\frac{1}{3}} \tag{6}$$

The above equations show that the classical weakly coupled plasma are generally dilute whereas the quantum weakly coupled plasma are found to be dense. Whether a system is classical or quantum is determined by the degeneracy parameter, $\chi = T_F/T$ and accordingly the Wigner formulation or the Vlasov formulation is used. The Wigner-Poisson (WP) formulation and Schrodinger-Poisson (SP) formulation are some of the mathematical models describing the properties of quantum plasma. The WP model is often used in the study of quantum kinetic behavior of plasma whereas the SP model describes the hydrodynamic behavior of plasma particles in quantum scales.

In early years of plasma physics, it was found that rewriting the non-linear quantum-like equation in the form of hydrodynamical equation can help in better physical understanding of it. The hydrodynamic equations essentially represent the densities and momentum evaluation of quantum particles. Bohm [7-9, 34, 35] and Madelung [36] carried out an elegant treatment by introducing an eikonal representation for the wave function evolution in the non-stationary Schrödinger equations. The quantum electron fluid equations were derived for the Klein-Gordon equations [37] and for the Dirac equation $\beta \to \infty$ s [38, 39] to incorporate quantum fluid formalism. The Madelung equations for quantum fluid are derived by [40, 41].

So far everybody has used the QHD model for extremely low temperatures. But in many cases such quantum effects are found with a finite temperature. Therefore, the formalism should be accordingly changed. A new model for non-linear quantum fluid equations at a finite temperature was given by Eliasson and Shukla [15]. This model, under specific limiting conditions of high and low temperature, converges to the regular equations. The model is explained in section 2 of the present paper. In this paper we have investigated arbitrary amplitudes Ion acoustic solitary structures. For small amplitude waves, the reductive perturbation technique (RBT) is generally used. But when the amplitude becomes large, this treatment is not valid. So some other mathematical method is to be used. The pseudopotential method of Sagdeev is one such approach.

The paper is organized in the following way. In section two the finite temperature model is introduced. Section three deals with the basic hydrodynamic model equations and derives the expression for Sagdeev's pseudopotential well. The next section is devoted to the study of solitary structures. Finally we discuss our results and come to a conclusion.

II. THE FINITE TEMPERATURE MODEL

We have based our mathematical model on a three-dimensional equilibrium state in which nonlinear plane electron plasma waves are propagating. For spin ½-particles i.e., Fermions, the 3D Fermi-Dirac equilibrium state is given by [42]

$$n_0 = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty \frac{E^{1/2}dE}{e^{\beta(E-\mu)}+1} = -\frac{1}{2\pi^2\beta^{3/2}}\left(\frac{2m}{\hbar^2}\right)^{3/2}\Gamma\left(\frac{3}{2}\right)Li_{3/2}\left(-e^{\beta\mu}\right) \tag{7}$$

where m is the electron mass, $\hbar$ is Planck's constant divided by 2π, $\beta = 1/k_B T_{e0}$, k_B is the Boltzmann constant, T_{e0} is the background temperature, μ is the chemical potential and $\mathrm{Li}_\nu(\xi)$ is the polylo $\mathcal{E}_F$ garithm function [43, 44]. $\Gamma(3/2) = \sqrt{\pi}/2$. Equation (7) gives the dependence of the equilibrium chemical potential μ on the temperature parameter β and on the equilibrium number density n_0. When, that is, in the limit of electron temperature, we have $\mu \to \mathcal{E}_F$ (is the Fermi energy). Thus, we have from equation (7)

$$n_0 = \frac{1}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\mathcal{E}_F^{\,3/2} \tag{8}$$

or $$\mathcal{E}_F = \left(3\pi^2 n_0\right)^{2/3}\frac{\hbar^2}{2m} \tag{9}$$

In plasmas which are free from collisions, plane longitudinal waves lead to adiabatic compression along one dimension only, and the plasma is heated adiabatically only in the velocity dimension along the wave propagation direction. This leads to temperature anisotropy of the electron distribution that varies with the wave motion. The phase fluid is incompressible in phase space in a classical Vlasov picture, and the Vlasov equation can be written as $\mathrm{d}f/\mathrm{d}t = 0$, showing that the distribution function f does not change its value along the particle trajectories. In a quantum picture, using the Wigner equation, the incompressibility of f is violated by quantum tunneling, but it can be assumed that the incompressibility of the electron phase fluid is true to first order. Now, based on these assumptions, we consider a non-equilibrium particle distribution function of the form

$$f(x,\mathbf{v},t) = \frac{\delta}{\exp\left\{\left(\frac{\beta m}{2}\right)\left[\left(v_x - v_{ex}\right)^2\eta + v_y^2 + v_z^2\right] - \beta\mu\right\}+1} \tag{10}$$

where δ is a normalization constant, $v_{ex}(x, t)$ is the mean velocity of the particles, and $\eta(x, t) = T_{e0}/T_{ex}(x, t)$ gives the temperature anisotropy of the distribution function (which is to be determined self-consistently from the number density variations). As a first approximation, it can be assumed that the chemical potential μ does not change during the non-equilibrium dynamics of the plasma. For a constant value of μ, it is seen from equation (10) that though f is deformed in phase space, its amplitude remains unchanged. The maximum value of f is always $f_{max} = \delta/[\exp(-\beta\mu) + 1]$. Normalizing f such that its integral over velocity space equals n_0 at equilibrium when $\eta=1$ and $v_{ex}=0$, we have

$$\delta = -\frac{n_0}{\mathrm{Li}_{3/2}\left(-\mathrm{e}^{\beta\mu}\right)}\left(\frac{\beta m}{2\pi}\right)^{3/2} \tag{11}$$

Comparing equation (11) with equation (7), we have δ = 2 $(m/2\pi\hbar)^3$. Now in order to prepare for our fluid treatment, we have calculated the zeroth, first and second moments of the distribution function f and they are given by:

$$n_{\mathrm{e}}\left(x,t\right) = \int f\,\mathrm{d}^3v = -\frac{\delta \mathrm{Li}_{3/2}(-\mathrm{e}^{\beta\mu})}{\eta^{3/2}}\left(\frac{2\pi}{\beta m}\right)^{3/2} = \frac{n_0}{\eta(x,t)^{1/2}} \tag{12}$$

or $\eta(x,t) = [n_0 / n_{\mathrm{e}}(x,t)]^2$, which gives the temperature anisotropy as a function of the number density variations,

$$\langle v_x \rangle = \frac{1}{n_{\mathrm{e}}}\int v_x f \mathrm{d}^3v = v_{\mathrm{ex}}(x,t) \tag{13}$$

and $\langle v_x^2 \rangle = \dfrac{1}{n_{\mathrm{e}}}\displaystyle\int v_x^2 f \mathrm{d}^3 v$

$$\begin{aligned} v &= v_{\mathrm{ex}}^2 + \frac{1}{n_{\mathrm{e}}}\int (v_x - v_{\mathrm{ex}})^2 f \mathrm{d}^3v \\ &= v_{\mathrm{ex}}^2 - \frac{2\pi\delta}{3\eta^{3/2}n_{\mathrm{e}}}\left(\frac{2}{\beta m}\right)^{5/2}\Gamma\left(\frac{5}{2}\right)\mathrm{Li}_{5/2}(-\mathrm{e}^{\beta\mu}) \\ &= v_{\mathrm{ex}}^2 + \frac{n_0}{\beta m n_{\mathrm{e}}}\frac{\mathrm{Li}_{5/2}(-\mathrm{e}^{\beta\mu})}{\mathrm{Li}_{3/2}(-\mathrm{e}^{\beta\mu})}\left(\frac{n_e}{n_0}\right)^3 \end{aligned} \tag{14}$$

where equations (11) and (12) have been used in the last step for δ and η respectively, and we have put $\Gamma(5/2) = 3\sqrt{\pi}/4$. Restricting to spatial dependence only along the x-direction, the Wigner equation gives

$$\frac{\partial f}{\partial t} + v_x\frac{\partial f}{\partial x} = -\mathrm{i}\frac{em^3}{(2\pi)^3\hbar^4}\iint \mathrm{d}^3\lambda \mathrm{d}^3v' \exp\left[\begin{array}{l}\left[i\dfrac{m}{\hbar}(\mathbf{v}-\mathbf{v}').\boldsymbol{\lambda}\right]\times \\ \left[\phi\left(x+\dfrac{\lambda_x}{2},t\right)-\phi\left(x-\dfrac{\lambda_x}{2},t\right)\right]\end{array}\right] f(x,\mathbf{v}',t) \tag{15}$$

where the Poisson equation has been used to determine the electrostatic potential ϕ

$$\frac{\partial^2\phi}{\partial x^2} = 4\pi e\left(\int f \mathrm{d}^3v - n_0\right). \tag{16}$$

The Wigner equation (15) converges to the Vlasov equation in the classical limit $\hbar \to 0$, and in the present case it is given by

$$\frac{\partial f}{\partial t} + v_x\frac{\partial f}{\partial x} = -\frac{e}{m}\frac{\partial\phi}{\partial x}\frac{\partial f}{\partial v_x} \tag{17}$$

The zeroth and first moments of the Wigner equation are given respectively by

$$\frac{\partial n_{\mathrm{e}}}{\partial t} + \frac{\partial(n_{\mathrm{e}}v_{\mathrm{ex}})}{\partial x} = 0 \tag{18}$$

And $$\frac{\partial(n_{\mathrm{e}}v_{\mathrm{ex}})}{\partial t} + \frac{\partial(n_{\mathrm{e}}\langle v_x^2\rangle)}{\partial x} - n_{\mathrm{e}}\frac{e}{m}\frac{\partial\phi}{\partial x} = 0. \tag{19}$$

Here, equation (14) gives the expression of $\langle v_x^2 \rangle$, using which we have

$$\frac{\partial(n_{\mathrm{e}}v_{\mathrm{ex}})}{\partial t} + \frac{\partial(n_{\mathrm{e}}v_{\mathrm{ex}}^2)}{\partial x} + n_0 v_{\mathrm{Te}}^2 G\frac{\partial}{\partial x}\left(\frac{n_{\mathrm{e}}}{n_0}\right)^3 - n_{\mathrm{e}}\frac{e}{m}\frac{\partial\phi}{\partial x} = 0 \tag{20}$$

where $v_{\mathrm{Te}} = (k_{\mathrm{B}}T_{\mathrm{e}}/m)^{1/2}$ is the electron thermal speed, and where we have introduced an electron degeneracy parameter G defined as

$$G \equiv \frac{\mathrm{Li}_{5/2}(-\mathrm{e}^{\beta\mu})}{\mathrm{Li}_{3/2}(-\mathrm{e}^{\beta\mu})} \tag{21}$$

It is an important parameter for the transition between the ultra-cold and the thermal cases. Using (18), we can rewrite equation (20) as

$$\frac{\partial v_{\mathrm{ex}}}{\partial t} + v_{\mathrm{ex}}\frac{\partial v_{\mathrm{ex}}}{\partial x} + \frac{n_0 v_{\mathrm{Te}}^2}{n_{\mathrm{e}}}G\frac{\partial}{\partial x}\left(\frac{n_{\mathrm{e}}}{n_0}\right)^3 - \frac{e}{m}\frac{\partial\phi}{\partial x} = 0 \tag{22}$$

That the plane wave propagation is essentially one-dimensional, with propagation along a single dimension and without energy exchange in other dimensions [15], is reflected by the exponent γ = 3 on the electron number density. This can be verified by putting D = 1 in the relation $\gamma = (D+2)/D$. It can be seen that equations (7) and (8) imply that

$$\left(\beta\mathcal{E}_{\mathrm{F}}\right)^{3/2} = -\frac{3\pi^{1/2}}{4}\mathrm{Li}_{3/2}(\text{-}\mathrm{e}^{\beta\mu}) \tag{23}$$

which can be used to find μ for different values of β and $\mathcal{E}_{\mathrm{F}}$. Figure 1 shows the variation of G with $\beta\mathcal{E}_{\mathrm{F}}$. For $\beta\mathcal{E}_{\mathrm{F}} \gg 1$, we get $\mu \simeq \mathcal{E}_{\mathrm{F}}$ and $G \simeq 2\beta\mathcal{E}_{\mathrm{F}}/5$, and for $\beta\mathcal{E}_{\mathrm{F}} \to 0$, we get

$\mu \to -\infty$ and $G \to 1$. For $\beta \mathcal{E}_F \lesssim 2$, it is a good approximation to use the formula $G \simeq 1 + (1/3\sqrt{2\pi})(\beta \mathcal{E}_F)^{3/2}$. This is obtained by a small argument approximation $\mathrm{Li}_\nu(\xi) \approx \xi + \xi^2/2^\nu$ of the polylogarithm function.

We note that in the absence of quantum diffraction effects, the zeroth and first moments of the Wigner equation (15) produce the same result as taking the zeroth and first moments of the Vlasov equation (17). So the formalism used here to derive the quantum field equations does not produce, in equation (22), the quantum diffraction term (Bohm-potential). This potential, however, can be included if we carefully consider the quantum diffraction effects as was done in the 1D, non-linear case by using a multi-stream model[15] and in the 3D, linear case by using a series of canonical transformations of the Hamiltonian[15]. We have assumed that the Bohm potential depends only on the mean distance between particles, and is independent of the thermal fluctuations in a finite temperature plasma (which is justified in our case), and thus we may postulate, without mathematical rigor, that the continuity and momentum equations for the electrons and ions take the form

$$\frac{\partial n_e}{\partial t} + \nabla.(n_e \mathbf{v}_e) = 0 \tag{24}$$

and

$$\frac{\partial \mathbf{v}_e}{\partial t} + \mathbf{v}_e \nabla \mathbf{v}_e + \frac{n_0 v_{Te}^2}{n_e} G \nabla \left(\frac{n_e}{n_0}\right)^3 - \frac{e}{m_e}\nabla\phi + \frac{\hbar^2}{2m_e}\nabla\left(\frac{1}{\sqrt{n_e}}\nabla^2\sqrt{n_e}\right) = 0 \tag{25}$$

respectively. The same for ions are given by

$$\frac{\partial n_i}{\partial t} + \nabla.(n_i \mathbf{v}_i) = 0 \tag{26}$$

$$\frac{\partial \mathbf{v}_i}{\partial t} + \mathbf{v}_i \nabla \mathbf{v}_i + \frac{e}{m_i}\nabla\phi + \frac{1}{m_i n_i}\nabla p_i + \frac{\hbar^2}{2m_i}\nabla\left(\frac{1}{\sqrt{n_i}}\nabla^2\sqrt{n_i}\right) = 0 \tag{27}$$

accordingly.

The last term in the L.H.S. of each of the equations (25) and (27) is the nonlinear quantum diffraction force. Applying closure property, we use the Poisson equation to close the system:

$$\nabla^2\phi = 4\pi e\left(\int f \mathrm{d}^3 v - n_0\right) \tag{28}$$

III. BASIC EQUATIONS

Our system contains homogeneous and unmagnetised electron-ion quantum plasma. In quantum plasma the effect of ion temperature on the ion acoustic wave (IAW) is studied by assuming the electrons to be inertialess & the ions are taken to be inertial. The phase velocity of the wave is taken to be $V_{Fi} << \omega / k << V_{Fe}$ (where V_{Fi} and V_{Fe} are the Fermi velocities of ions and electrons respectively). Ion pressure effects due to ion Fermi temperature can therefore be ignored. The 1-D basic dynamic equations in the unnormalised form, ignoring the non-linear mechanisms of ion acoustic waves in quantum plasmas obtained from section two (The finite temperature Fermi plasma model) and neglecting quantum diffraction effects for ions, are given as;

$$\frac{\partial n_e}{\partial t} + \frac{\partial (n_e v_e)}{\partial x} = 0 \tag{29}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_i)}{\partial x} = 0 \tag{30}$$

$$0 = \frac{e}{m_e}\frac{\partial \phi}{\partial x} - \frac{n_0 v_{Te}^2}{n_e} G \frac{\partial}{\partial x}\left(\frac{n_e}{n_0}\right)^3 + \frac{h^2}{2m_e^2}\frac{\partial}{\partial x}\left(\frac{\partial^2 \sqrt{n_e} / \partial x^2}{\sqrt{n_e}}\right) \tag{31}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{e}{m_i}\frac{\partial \phi}{\partial x} - \sigma_1 n_i \frac{\partial n_i}{\partial x} \tag{32}$$

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi e (n_i - n_e) \tag{33}$$

Here n_j, v_j, m_j, $-e$ are the density, velocity field, mass, and charge, respectively where j $= e$, i stands for electrons and ions. Meanwhile, $\hbar = h/2\pi$ is the reduced Planck constant, ϕ is the electrostatic wave potential, p_e is the electron pressure, and $\sigma_1 = 3[T_{Ei}/T_{Fe}]$ is the ion-to-electron Fermi Temperature ratio, where T_{Fj} is the Fermi temperature of the j^{th} species. At equilibrium, we have $n_{i0}=n_{e0}=n_0$. We also assume that the ions behave as a one dimensional Fermi gas at zero temperature and therefore the pressure law [45] is:

$$p_i = \frac{m_i V_{Fi}^2}{3 n_{i0}^2} n_i^3 \tag{34}$$

where m_i is the mass of ions; $V_{Fi} = \sqrt{2k_B T_{Fi}/m_i}$ is the Fermi thermal speed, T_{Fi} is the Fermi temperature and k_B is the

Boltzmann constant; n_i is the number density with the equilibrium value n_{i0} .Now using the following normalization $x \to \frac{x\omega_i}{c_s},\ t \to t\omega_i,\ \phi \to \frac{e\phi}{2k_B T_{Fe}}, n_j \to \frac{n_j}{n_0},\ u_j \to \frac{u_j}{c_s}$ the normalized set of equations;

$$\frac{\partial n_e}{\partial t} + \frac{\partial (n_e v_e)}{\partial x} = 0 \tag{35}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_i)}{\partial x} = 0 \tag{36}$$

$$0 = \frac{\partial \Phi}{\partial x} - 3G\alpha^2 n_e \frac{\partial n_e}{\partial x} + \frac{H^2}{2}\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_e}}\partial^2 \sqrt{n_e} \Big/ \partial x^2\right) \tag{37}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{\partial \Phi}{\partial x} - \sigma_1 n_i \frac{\partial n_i}{\partial x} \tag{38}$$

$$\frac{\partial^2 \phi}{\partial x^2} = (n_i - n_e) \tag{39}$$

in which $\omega_e = \sqrt{4\pi n_{e0} e^2 / m_e}$ is the plasma frequency, $c_s = \sqrt{2k_B T_{Fe} / m_e}$ is the quantum ion-acoustic speed. H is the non-dimensional quantum diffraction parameter defined as $H = \hbar\omega_{ec} / 2k_B T_{Feh}$, where T_{Fe} is the Fermi temperatures for electrons.

In order to get localized stationary solution, let us assume that all dependent variables are functions of single independent variable:

$$\xi = x - Mt \tag{40}$$

where M is the Mach number defined by v/c_s, v is the velocity of the nonlinear waveform moving with the frame.

By integrating (37) once and applying boundary conditions $n_e \to 1$ & $\phi \to 0$ at $\xi = |\pm\infty|$; we obtain:

$$\phi = -\frac{3}{2}G\alpha^2 + \frac{3}{2}n_e^2 - \frac{H^2}{2}\frac{1}{\sqrt{n_e}}\frac{\partial^2 \sqrt{n_e}}{\partial x^2} \tag{41}$$

From the ion continuity equation (36) and ion momentum equation (38) with proper boundary conditions $\phi \to 0,\ v_i \to 0,\ n_i \to 1$ as $\xi \to |\pm\infty|$ we obtain:

$$v_i = M(1 - \frac{1}{n_i}) \tag{42}$$

$$v_i^2 - 2Mv_i + \sigma_1 n_i^2 - \sigma_1 = -2\phi \tag{43}$$

Substituting equation (42) in (43) we get,

$$\phi = \frac{M^2}{2}[1 - \frac{1}{n_i^2}] + \frac{\sigma_1}{2}(1 - n_i^2) \tag{44}$$

Now by employing quasi-neutrality conditions $n_i \approx n_e = n$ (45)

and also substituting $z = \sqrt{n}$, from equations (41-44) we obtain

$$\frac{H^2}{2}\frac{\partial^2 Z}{\partial \xi^2} = -\frac{3G\alpha^2}{2}Z + \frac{3G\alpha^2}{2}Z^5 - \frac{M^2}{2}[Z - \frac{1}{Z^3}] - \frac{\sigma_1}{2}[Z - Z^5] \tag{46}$$

Multiplying both sides of equation (46) by $dz/d\xi$ and integrating with the boundary condition $n'' \to 0$ and $n' \to 0$ and $n \to 0$, (where primes represent derivatives with respect to ξ) we obtain the nonlinear differential equation in terms of density as:

$$\frac{1}{2}\left(\frac{dn}{d\xi}\right) + u(n) = 0 \tag{47}$$

where, the Sagdeev's pseudopotential is defined as equation:

$$U(\mathrm{x}) = \frac{8n}{H^2}\left[\begin{array}{l}\left(\frac{3G\alpha^2}{2} + M^2 + \sigma_1\right)\left(\frac{n-1}{4}\right) - \\ \left(\frac{G\alpha^2}{2}\right)\left(\frac{n^3 - 1}{2}\right) + \frac{M^2}{4}\left(\frac{1}{n} - 1\right) - \frac{\sigma_1}{12}\left(n^3 - 1\right)\end{array}\right] \tag{48}$$

Equation (48) is called the energy integral of an oscillatory particle of mass unity moving with a velocity $n' = dn/d\xi$ at position n in a potential well U(n). It has quite similar expression as found by Chatterjee *et al.* [46]. If the ion temperature is neglected the equation (48) agrees with equation (19) in the article investigated by Mahmood & Mustaque [47].

The dependence of the pseudopotential U(n) on electron degeneracy parameter (G), quantum diffraction parameter (H), ion-to-electron Fermi temperature ratio (σ) and Mach number (M) is shown in figures 1-4. In figure 1, it is found that with increasing value of G, the dip of the pseudopotential well increases (i.e. it becomes more negative). Figure 2 shows the dependence of U(n) on quantum diffraction parameter H. The potential well becomes less deep. The value of one of the roots, n_m, lies between n = 0.6 to 0.7. From 0 to n_m, the slope of the pseudopotential curve is less.

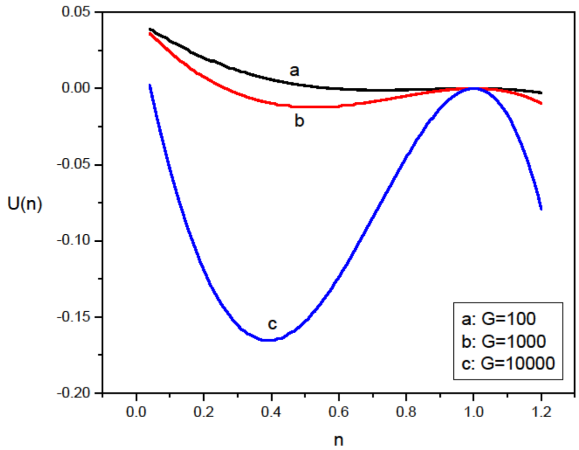

Fig. 1: U (n) is plotted vs. *n* for different values of electron degeneracy parameter G; other parameters are *M*=0.6, *H*=4 and σ=0.2.

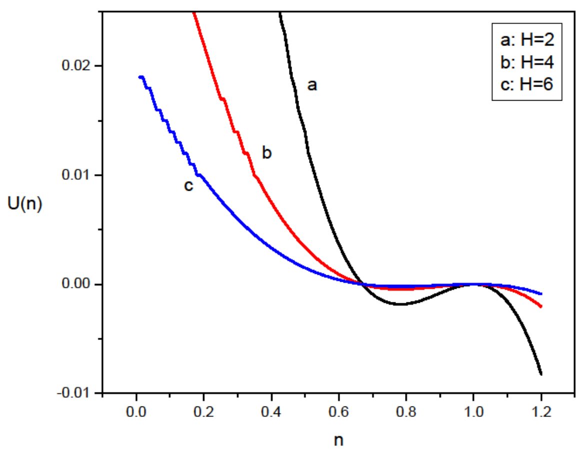

Fig. 2: U (n) is plotted vs. *n* for different values of Quantum diffraction parameter H; other parameters are *M*=0.6, *G*=100 and σ=0.2.

Figure 3 depicts the dependence of U(n) on ion-to-electron Fermi temperature ratio σ. It is found that with the increasing value of σ, the potential well becomes slightly deeper. The dependence on Mach number is shown in figure 4. It only changes the value of n_m.

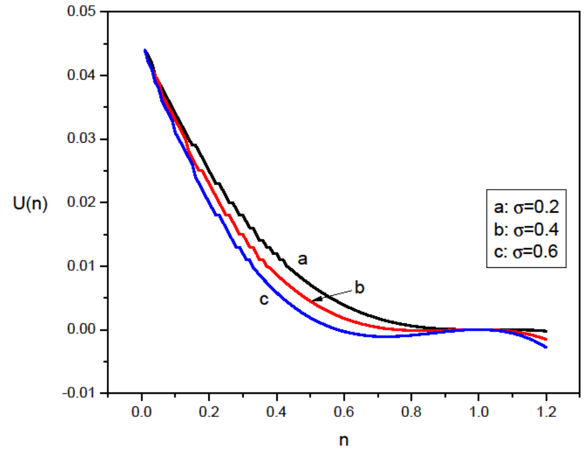

Fig. 3: U (n) is plotted vs. *n* for different values of ion-to-electron temperature ratio σ; other parameters are *M*=0.6, *G*=100 and H=4.

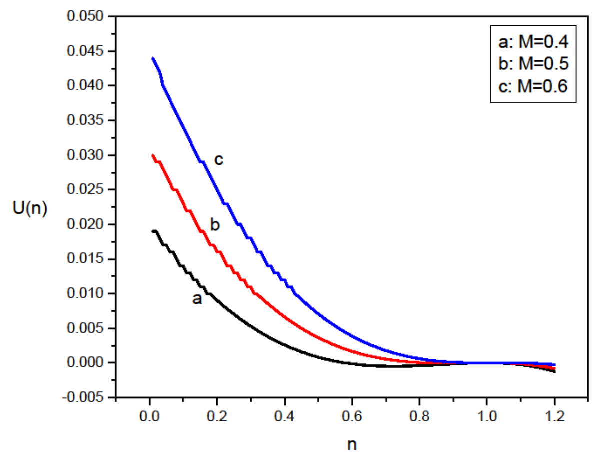

Fig. 4: U (n) is plotted vs. *n* for different values of Mach number M; other parameters are σ =0.2, *G*=100 and H=4.

IV. SOLITARY WAVE SOLUTIONS

Equation (48) describes the Sagdeev's pseudopotential well *U*(*n*), in which the particle's motion is executed, as a function of n. The characteristics of the pseudopotential *U*(*n*) will then decide the conditions for the existence of solitary wave solution. If it is found that between any two roots (in this case, 0 and n_m) of the pseudopotential, *U*(*n*) is negative, then an oscillatory wave is found. On the contrary, if in the interval one root is a single root and another is a double root, then a solitary wave can be predicted. If both the roots are double root, then a double layer exists. The initial conditions are chosen such that the double root appears at n=1. Therefore it takes an infinitely long time to get away from it and n reaches a minimum at n_m, then again taking infinitely long time to return to n=0. Hence, the conditions for the existence of soliton solution are the following:

a) $U(n)=0$ at n=1 and n=n_m (49a)

b) $\dfrac{dU(n)}{dn}=0$ at n=1 but $\dfrac{dU(n)}{dn}\neq 0$ at $n=n_m$ (49b)

c) $\dfrac{d^2U(n)}{dn^2}<0$ at n=1 (49c)

If n_m is less than one then rarefractive solitary wave structures are formed. On the other hand if it is greater than unity, then compressive structures are obtained. It is to be noted that complex U(n) is not physically allowed as it would imply complex density which is not physical. From equation (48), it is seen that the shape of the solitary structures can be determined from the following relation:

$$\xi=\pm\int_{n_m}^{n}\frac{dn}{\sqrt{-2U(n)}} \tag{50}$$

Figures 5-8 show the solitary profile structures with variations in G, H, σ and M. In figure 5, it is clearly depicted that as G increases, the width of the soliton decreases and its amplitude increases.

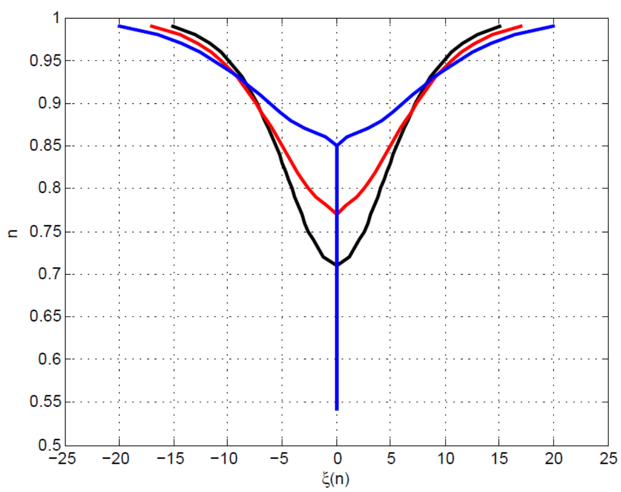

Fig. 5: *n* is plotted vs. ξ with variation of electron degeneracy parameter G. The blue curve denotes G=100, the red curve denotes G=1000, the black curve denotes G=10000; other parameters are *M*=0.6, H=2 and σ=0.2.

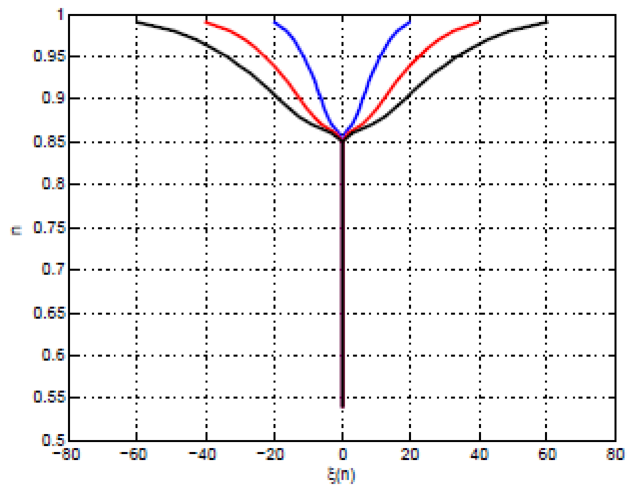

Fig. 6: *n* is plotted vs. ξ with variation of quantum diffraction parameter H. The blue curve denotes H=2, the red curve denotes H=4, the black curve denotes H=6. Other parameters are *M*=0.6, G=100 and σ=0.2.

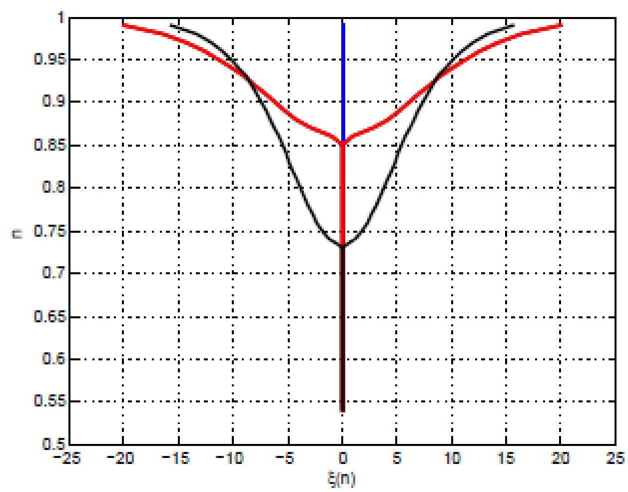

Fig. 7: *n* is plotted vs. ξ with variation of ion-to-electron temperature ratio σ. The blue curve denotes σ =0.2, the red curve denotes σ =0.4, the black curve denotes σ =0.6; other parameters are *M*=0.6, H=2 and G=100.

On the contrary, as quantum diffraction effects become more prominent, the solitons become wider but there is no significant effect on the amplitude (figure 6). In figure 7, it is found that as the value of ion-to-electron Fermi temperature ratio increases, the amplitude increases and the width decreases, which is directly opposite to the effect of electron degeneracy parameter (G). Finally figure 8 shows that the change in Mach number has a very small effect on the properties of ion acoustic solitons; the width decreases slightly but the amplitude remains constant.

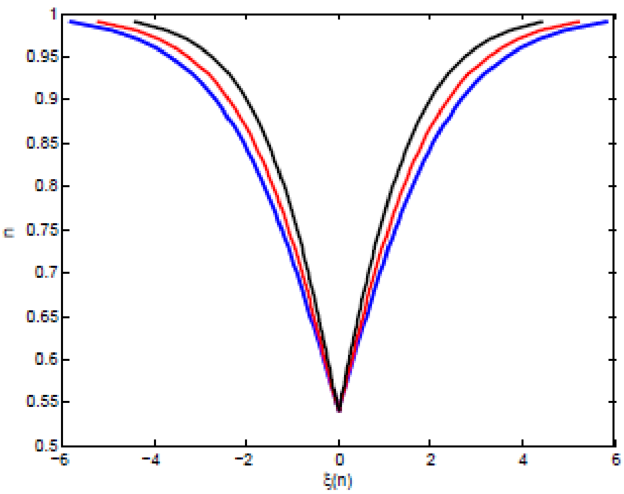

Fig. 8: *n* is plotted vs. ξ with variation of Mach number M. The blue curve denotes M = 0.4, the red curve denotes M =0.5, the black curve denotes M =0.6; other parameters are σ =0.2, G=100 and H=2.

V. CONCLUSION & REMARKS

The paper reports on the formation and properties of large amplitude ion acoustic solitary structures in finite temperature degenerate Fermi plasma by employing Sagdeev's pseudopotential approach. The effects of electron degeneracy parameter (G), quantum diffraction parameter (H), ion-to-electron Fermi temperature ratio (σ) and Mach number (M) are studied with great detail. The results obtained here are important in further studies of quantum plasma at non-zero temperature.

ACKNOWLEDGEMENTS

The authors would like to thank Techno India University, Saltlake, Kolkata and Jadavpur University, Kolkata for providing library and laboratory facilities as well as access to some journals. They would further extend their gratitude to Agniv Chandra for providing assistance during the programming for the plots and Prof. Basudev Ghosh for providing valuable insight into the subject.

REFRENCES

[1] Chabrier, G., Douchin, F., Potekhin, A. Y., "Dense astrophysical plasmas" (2002), Journal of Physics of condensed matter, 149, 133.
[2] Barnes, W. L., Dereux A., Ebbesen, T. W, "Surface plasmon subwavelength optics" (2003), *Nature(London)*, 424, 824
[3] Becker, K., Koutsospyros, A., Yin, S. M., Christodoulatos, C., Abramzon, N., Joaquin, J. C., Brelles-Marino, G., "Environmental biological applications of microplasmas" (2005), Plasma Physics and Controlled Fusion 47, B513.
[4] Ang, L. K., Kwan, T. J. T., Lau, Y. Y., "New Scaling of Child-Langmuir Law in the Quantum Regime" (2003), Physical Review Letters, 91, 208303.
[5] Jung, Y. D., "Quantum-mechanical effects on electron–electron scattering in dense high-temperature plasmas" (2001), Physics of Plasmas, 8, 3842.
[6] L. K. Ang, Y. Y. Lau, and T. J. T. Kwan, "Simple Derivation of Quantum Scaling in Child–Langmuir Law" (2004), IEEE Transactions on Plasma Science 32(2), 410.
[7] Bohm, D., "A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. I" (1952), Phys. Rev. 85 166.
[8] Bohm, D., "A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. II" (1952), Phys. Rev. 85, 180
[9] Bohm, D., Pines, D., "A Collective Description of Electron Interactions: III. Coulomb Interactions in a Degenerate Electron Gas" (1953), Phys. Rev. 92 609.
[10] Haas, F., "*Quantum Plasmas: A Hydrodynamic Approach"* (2011), Springer.
[11] Shukla, P. K., Eliasson, B., "Nonlinear aspects of quantum plasma physics" (2010), Physics-Uspekhi. 53, 51.
[12] Manfredi, G., "How to model quantum plasmas" (2005), Fields Institute Communications. 46, 263–287.
[13] Brodin, G., Marklund, M., Eliasson, B., Shukla, P. K., "Quantum-Electrodynamical Photon Splitting in Magnetized Nonlinear Pair Plasmas" (2007). Physical Review Letters, 125001, 98.
[14] Brodin, G., Marklund, M., "Spin magneto-hydrodynamics" (2007) New Journal of Physics, 9 (8), 277.
[15] Eliasson, B., Shukla, P. K., "Nonlinear quantum fluid equations for a finite temperature Fermi plasma" (2008), Physica Scripta, 78, 025503.
[16] Misra, A. P., Bhowmik, C., "Nonplanar ion-acoustic waves in a quantum plasma" (2007) Physics Letters. A, 369, 90.
[17] Chatterjee, P., Roychoudhury, R., "Effect of ion temperature on large-amplitude ion-acoustic solitary waves in relativistic plasma" (1994), Physics of Plasmas 1, 2148.
[18] Chatterjee, P., Das, B., "Effect of electron inertia on the speed and shape of ion-acoustic solitary waves in plasma" (2004), Physics of Plasmas 11, 3616.
[19] Ghosh, B., Chandra, S, Paul, S.N, "Amplitude Modulation of Electron Plasma waves in a quantum plasma " (2011), Physics of Plasmas,012106,18.
[20] Ghosh, B., Chandra, S, Paul, S.N, "Relativistic Effects on the modulational instability of electron plasma waves in a quantum plasma" (2011), Pramana-Journal of Physics, 78,779-790.
[21] Chandra, S., Ghosh, B., Paul, S.N, "Linear Nonlinear propagation of electron plasma waves in quantum plasma" (2012), Indian Journal of pure applied physics, NISCAIR 50,314.
[22] Chandra, S., Ghosh, B., "Modulational instability of electron plasma in finite temperature quantum plasma" (2012), World academy of science engineering and technology, 6 (11), 576.

[23] Chandra, S., Ghosh, B., "Propagation of electron acoustic solitary waves in weakly relativistically degenerate quantum plasma", (2013), World academy of science engineering technology, 7(3), 614.
[24] Chandra, S., Ghosh, B, "Nonlinear electrostatic wave structure in quantum plasma including finite temperature relativistic effects" (2013), Indian Journal of pure applied physics, NISCAIR 51, 627.
[25] Chandra, S., Ghosh, B., "Modulational Instability of electron-acoustic waves in a relativistically degenerate quantum plasma" (2012), Astrophysics and Space Sciences, SPRINGER, 342,417-424.
[26] Chandra, S., Ghosh, B., "Nonlinear Solitary Structures of Electron Plasma Waves in a Finite Temperature Quantum Plasma" (2012), World Academy of Science Engineering. And Technology., Vol 6, No. 10, 1094
[27] Chandra, S., Ghosh, B., "Relativistic effects on the nonlinear propagation of electron plasma waves in dense quantum plasma with arbitrary temperature" (2012) Int. J. of Engineering Research and Development Vol 3, Issue 1, pp-51-57.
[28] Chandra, S., Ghosh, B., "Nonlinear Surface Waves on a Quantum Plasma Half-Space with Arbitrary Temperature"2013), (International Journal of Systems, Algorithms & Applications, 3, ICASE, 1-3
[29] Chandra, S., Ghosh, B., "Finite Temperature Effects on the Linear Dispersion Properties of Electron-Acoustic Waves in Degenerate Plasma" (2013) International Journal of Systems, Algorithms & Applications, 3, ICASE 4-5
[30] Chandra, S., Maji, P., Ghosh, B., Propagation of Nonlinear Surface Waves in Relativistically Degenerate Quantum Plasma Half-Space, (2014) , World Academy of Sci. Engg. And Tech., Vol:8 No:5, 835
[31] Chandra, S., Ghosh, B., "Harmonic Generation by Electrostatic Surface Waves on a Quantum Plasma Half- Space" (2012), (Proceedings of PLASMA, P.U)
[32] Roychoudhury, R., Bhattacharyya, S., "Ion-acoustic solitary waves in relativistic plasmas" (1987), Physics of Fluids 30, 2582.
[33] Sahu, B., Roychoudhury, R., "Quantum ion acoustic shock waves in planar nonplanar geometry" (2007) Physics of Plasmas 14, 072310.
[34] Bohm, David, Hiley, Basil, "*The Undivided Universe: An Ontological Interpretation of Quantum Theory*" Routledge, 1993, ISBN 0-415-06588-7, therein Chapter 3.1. *The main points of the causal interpretation*, p. 22–23
[35] Bohm, D., Hiley, B. J., "*On the intuitive understanding of nonlocality as implied by quantum theory*" (1975), Foundations of Physics, Volume 5, Number 1, pp. 93-109, doi:10.1007/BF01100319
[36] Madelung, E Z., "Quantum Theory in Hydrodynamical Form" (1927), Phys. 40, 322
[37] Takabayashi, T., "On the formulation of quantum mechanics associated with classical pictures" (1952), Proceeding of Theoretical Physics. 8 143.
[38] Gindensberger, E., Meier, C., Beswick, J.A., "*Mixing quantum and classical dynamics using Bohmian trajectories*" 1 December 2000, Journal of Chemical Physics, vol. 113, no. 21, pp. 9369–9372
[39] Takabayashi, T., "Relativistic hydrodynamics of the Dirac matter", (1957), Supplement to Proceeding of Theoretical Physics. 4, 1-80
[40] Ghosh, S K., Deb, BM., "Quantum fluid dynamics of many electron systems in three dimensional space" (1982) International Journal of quantum chemistry, 22, 871-888
[41] Hiley, B. J., Greenberger, D., et al. "*Active Information and Teleportation*" (1999) p. 7; appeared in: Epistemological and Experimental Perspectives on Quantum Physics, et al. (eds.), pages 113-126, Kluwer, Netherlands
[42] Bransden B H and Joachain C J 2000 *Quantum Mechanics* (San Francisco: Benjamin/Cummings Pub. Co.)
[43] Abramowitz M and Stegun I A (eds) 1972 *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables* (New York: Dover)
[44] Lewin L, "*Polylogarithms and Associated Functions"* (1981) (New York: North-Holland) ISBN 0-444-00550-1
[45] Haas, F., Garcia, L. G., Goedert, J., Manfredi, G., "Quantum ion-acoustic waves" (2003), Physics of Plasmas 10, 3858.
[46] Chatterjee, P., Roy, K., Muniandy, S.V., Yap, S.L., Wong, C. S., "Effect of ion temperature on arbitrary amplitude ion acoustic solitary waves in quantum electron-ion plasma" (2009), Physics of Plasmas 16, 042311.
[47] Mahmood, S., Mustaque, A., "Quantum ion acoustic solitary waves in electron–ion plasmas: A Sagdeev potential approach" (2008) Physics Letters A 372, 3467.